THE BATTLE OF LEUCTRA

After having briefly introduced Game Theory...

The *battle of Leuctra* was a major skirmish between the Thebans and the Spartans that occurred in 371 BC.
It is remembered on the one hand because the Theban victory shattered Sparta's immense influence over the Greek peninsula just a generation after its victory in the Peloponnesian war, and on the other hand due to the innovative approach to tactics and armament the battle presented (especially on the Theban side) compared to earlier skirmishes.
The Spartans were in the habit to line up to face an enemy. On the contrary Epaminondas, the commander of the Theban army, decided to take position with the front diagonally, concentrating the army on the left side.
By seeing this deployment Cleombrotus, the commander of the Spartan army, had two possibilities: attacking directly by advancing simultaneosly, or encircling the enemy by moving the central formations backwards.
The opposite side sees Epaminondas having to choose between attacking directly, and attacking with an incursion seeking to break through the lines.
Since the two choices have to be basically taken at the same time and unbeknownst to each other, the situation can be modeled with the following table:

E can attack directly	E attacks directly	E attacks with an incursion
C attacks directly	2, 5	3, 2
C encircles the enemy	5, 1	4, 4

With minor modifications to the setting, it would have been possibile that Epaminondas were unable to attack directly, but able to perform an outflanking.
This maneuver consists in trying to hit the opposing army from a different direction, or from a more advantageous side for the attacker, avoiding clashing with the defense's strongest part.
Note that, despite an apparent similarity, outflanking is different than encircling. In the case, the table can be set up as follows:

E can outflank	E outflanks	E attacks with an incursion
C attacks directly	1, 1	3, 2
C encircles the	2, 5	4, 4

enemy		

Task 1: try to play in both situations. Which outcome will result by playing? (*use of Wooclap for data collection*)

Introduction of Nash equilibria, then Task 1 is performed again, in order to detect possible differences in light of having introduced that theoretical tool.

Task 2: in both the previous situations, Cleombrotus would have proceeded with the same choice. Create a context where for him would have been convenient to proceed with the other choice, by both assigning proper values to the payoff matrix, and motivating why they would hold.

Introduction of Pareto optimalities.

Task 3: the following systems of interactive components allows for generalizing the payoffs:

	E uses an alternative approach	E attacks with an incursion
C attacks directly	$C =$ 2 , $E =$ 5	$C =$ 4 , $E =$ 4
C encircles the enemy	$C =$ 1 , $E =$ 1	$C =$ 3 , $E =$ 2

Compute! Rese...

• C attacks directly, E uses an alternative approach is a Nash equilibrium and is Pareto optimal.

• C attacks directly, E attacks with an incursion is not a Nash equilibrium and is Pareto optimal.

• C encircles the enemy, E uses an alternative approach is not a Nash equilibrium

and is not Pareto optimal.

• C encircles the enemy, E attacks with an incursion is not a Nash equilibrium and is not Pareto optimal.

Try to vary the payoffs according to other possible contexts, and comment the results.

Task 4: let us suppose now that Epaminondas had the possibility to choose between attacking with an incursion and an alternative approach (neither attacking directly, nor outflanking) resulting in the following table:

	E uses an alternative approach	E attacks with an incursion
C attacks directly	3, 4	3, 2
C encircles the enemy	1, 1	4, 4

Which outcome will result by playing? (*possible use of Wooclap for data collection*)

Introduction of mixed Nash equilibria, then Task 4 is performed again, in order to detect possible differences in light of having introduced that theoretical tool.

Task 5: the following systems of interactive components allows for generalizing the payoffs:

	E uses an alternative approach	E attacks with an incursion
C attacks directly	$C =$ [ ] , $E =$ [ ]	$C =$ [ ] , $E =$ [ ]
C encircles the enemy	$C =$ [ ] , $E =$ [ ]	$C =$ [ ] , $E =$ [ ]

Compute! Rese...

Let us suppose that E "uses an alternative approach" with probability *p*; then, C expects to obtain _____ by "attacking directly", _____ by "encircling the enemy".

Player C would choose randomly if they are equal, that is, if the first degree equation _____ is verified.

Its solution, that is the probability that E "uses an alternative approach" in a mixed strategy equilibrium, is _____ .

Analogously, let us suppose that C "attacks directly" with probability *q*; then, E expects to obtain _____ by "using an alternative approach", _____ by "attacking with an incursion".
Player E would choose randomly if they are equal, that is, if the first degree equation _____ is verified.
Its solution, that is the probability that C "attacks directly" in a mixed strategy equilibrium, is _____ .

Try to vary the payoffs according to a proper context, and comment the results.

SOLUTIONS

Task 1: the only Nash equilibrium in the first table (E can attack directly) is "C encircles the enemy, E attacks with an incursion", with payoffs (4,4).
The only Nash equilibrium in the second table (E can outflank) is "C encircles the enemy, E uses an alternative approach", with payoffs (2,5).

Task 2: by swapping the rows in one of the two tables, the only Nash equilibrium would require C to attack directly.

A context can involve E having available two strategies different from those presented in the tables, resulting in such payoffs.

Task 4: the game possesses the two (pure) Nash equilibria "C attacks directly, E uses an alternative approach" (with payoffs 3,4) and "C encircles the enemy, E attacks with an incursion" (with payoffs 4,4)
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As in the battle of sexes, there is not a rational reason to prefer one over them.
It is also present a mixed Nash equilibrium, where E has probability to use an alternative approach of 1/3, while C has probablity to attack directly of 3/5 (the other probabilities are complementary).