DIGITAL MATHEMATICS APPLIED IN DEFENCE AND SECURITY EDUCATION (DIMAS)

KA220-HED - Cooperation Partnerships in Higher Education

2023-1-BG01-KA220-HED-000156664

Educational resource

(A3.2)

May 2025

SCENARIO No 3.6

Title	**Shortest Route Problem**
Short description	In many problems—such as material transfer, routing, and networking—graphs (directed or undirected) are used to represent the system's characteristics and data. These graphs consist of two main components: **nodes** and **edges**. **Nodes** represent entities, which may vary depending on the context of the problem. For example, nodes can represent road intersections, cities, telecommunication centers, production sites, distribution hubs, warehouses, and more. Each node is typically depicted as a circular or square shape, containing a unique identifier such as a number, letter, or name. **Edges** are lines that connect the nodes, representing the relationships between them. In most cases, only a single edge is defined between any two nodes. The interpretation of these edges depends on the specific problem: they may represent roads, transportation routes, communication cables, power lines, data links, pipelines, or similar connections. Thus, the connection of nodes via edges can signify the movement of goods, data transfer, the flow of electricity or fluids, or personnel movement. The core problem can be defined as follows: Determine the shortest route between two nodes, namely a starting point and a destination. The shortest route is the one with the lowest total cost, which could reflect distance, time, expense, or another relevant metric. Importantly, the shortest path does not need to include all the nodes in the graph.
Topics Involved	**Logistics for Defence and Security**
Areas of Mathematics	• Boolean Algebra • Mathematical Programming
Digital Mathematics Tools	Microsoft Excel – Solver tool
Learning Outcomes (knowledge, Skills, Responsibility & Autonomy)	**Knowledge:** Students know to • define the appropriate Boolean variables • determine the objective function • identify the restrictions **Skills:** After passing through this scenario, the students will be able to: • model and simulate any shortest route problem, • use software to solve the shortest route problem, • work in a team to solve a problem,

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- can prepare a note for the problem and give a short presentation on the solution.

Responsibility and Autonomy:
1. Students are ready to critically evaluate his/her knowledge and recognise the importance of knowledge in solving such problems
2. Take initiative and responsibility to propose an optimal solution to the shortest route problem.
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<tr><td>Methodologies adopted</td><td>Innovative teaching solutions will be used, such as specialized software to solve the shortest route problem using the MS Excel Solver tool, as an element of decision-making support.</td></tr>
<tr><td>Prerequisites</td><td>

- Mathematics: Boolean Algebra
- Mathematical Programming
- Microsoft Excel
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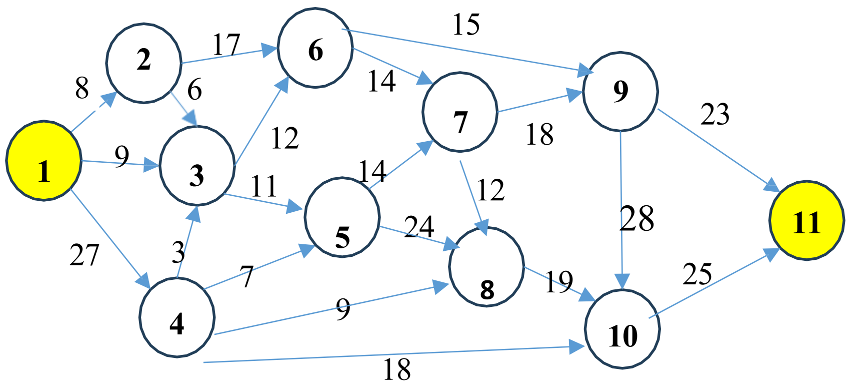
<tr><td>Estimated time</td><td>8 hours (including self-studies and syndicate work)</td></tr>
<tr><td>Task for students</td><td>

During a military operation, reinforcements (personnel, ammunition) must be transported from a starting point/node (1) to the final point/node of destination (11). The following oriented graph presents all possible routes (there are intermediate villages, towns) with the corresponding cost.



TASK 1:
1.1. Model the above problem. Particularly:
- Define the correct Boolean variables
- Determine the objective function
- Identify and write all the restrictions

1.2. Solve it using *MS Excel – Solver tool*

TASK 2:
Following the initial shipment, there is a need to deploy medical personnel and supplies to the destination at Point (11). At the same time, intelligence reports indicate the presence of enemy forces at Point (6). Solve it using *MS Excel – Solver tool*
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Assessment	Observation: Students are evaluated during the elaboration phase to document their understanding of modelling and solving the shortest route problem. Project: Individual projects. Final presentation. Complete initial and final feedback.

Solution

Task 1.1

Concerning the variables: Let x_{ij} be binary decision variables (Boolean variables) that indicate whether the connection from point i to point j is activated. Specifically, x_{ij}=1 if the route from point i to point j is realized, and x_{ij}=0 otherwise.
The *objective function* minimizes the total transfer cost across the network, computed as the sum of the costs c_{ij} associated with each activated route x_{ij} between points i and j. Which corresponds mathematically to:

$$\min K = \sum_{i=1}^{10} \sum_{j=2}^{11} c_{ij} \cdot x_{ij}$$

i.e.

$$K = 8x_{12} + 9x_{13} + 27x_{14} + 17x_{26} + 6x_{23} + 12x_{36} + 11x_{35} + 3x_{43} + \cdots +$$

$$19x_{810} + 23x_{911} + 28x_{910} + 25x_{1011}$$

The constraints must first ensure that the reinforcements originate from point 1 and ultimately reach the destination point 11.

$x_{12} + x_{13} + x_{14} = 1$ (departure from node 1)

$x_{910} + x_{1011} = 1$ (arrival at node 11)

Additionally, for any intermediate point i∈{2,…,10} visited by the reinforcements, the model must enforce that if they arrive at point i, they must also depart from it, continuing their progression toward point 11.

Point 2: $x_{12} = x_{23} + x_{26}$, Point 3: $x_{13} + x_{23} + x_{43} = x_{35} + x_{36}$

Point 4: $x_{14} = x_{43} + x_{45} + x_{48} + x_{410}$, Point 5: $x_{35} + x_{45} = x_{57} + x_{58}$

Point 6: $x_{26} + x_{36} = x_{67} + x_{69}$, Point 7: $x_{67} + x_{57} = x_{78} + x_{79}$

Point 8: $x_{48} + x_{58} + x_{78} = x_{810}$, Point 9: $x_{69} + x_{79} = x_{910} + x_{911}$

Point 10: $x_{410} + x_{810} + x_{910} = x_{1011}$

Task 1.2:

See the attached Excel file

Using an MS Excel File, enter in cells A1 to U1 the variables x12 to x1011 as the figure below. In the corresponding cells A2 to U2, the values of the corresponding variables will be displayed.

In cell D6, please enter the variable Z, while in cell D7, enter the following function

D7=8*A2+9*B2+27*C2+6*D2+17*E2+11*F2+23*G2+3*H2+7*I2+9*J2+18*K2+14*L2+24*M2+14*N2+15*O2+12*P2+18*Q2+19*R2+28*S2+23*T2+25*U2

In cells G7 to G17, please enter the restrictions as shown below:

"G7" = A2+B2+C2

"G8" =T2+U2

"G9" = A2

"G10" = B2+D2+H2

"G11" = C2

"G12" =F2+I2

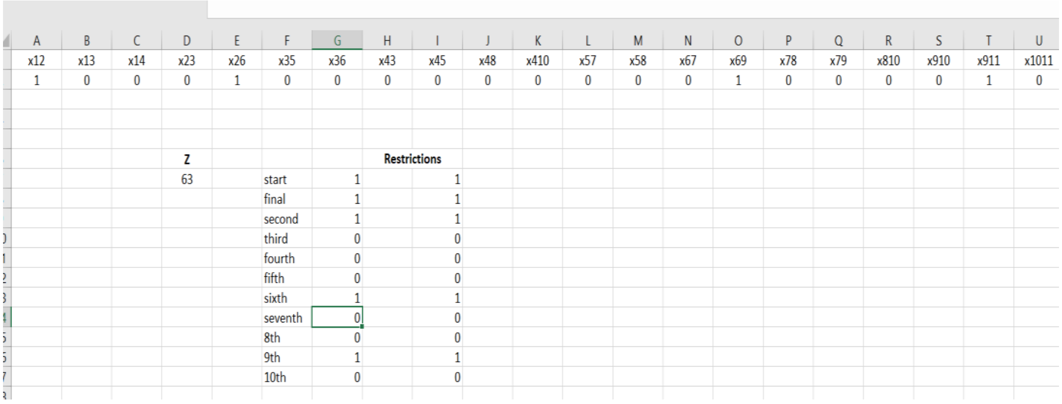
"G13" = E2+G2

"G14" = N2+L2

"G15" = J2+M2+P2

"G16" = O2+Q2

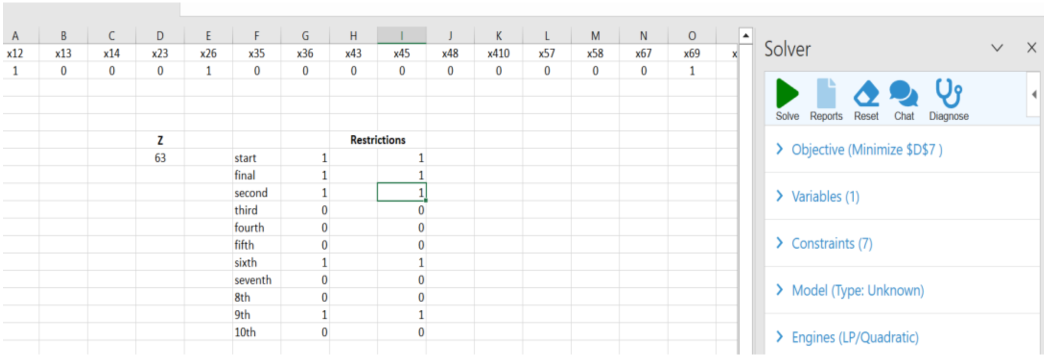
"G17" = K2+R2+S2



x12	x13	x14	x23	x26	x35	x36	x43	x45	x48	x410	x57	x58	x67	x69	x78	x79	x810	x910	x911	x1011
1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0

Z		Restrictions	
63	start	1	1
	final	1	1
	second	1	1
	third	0	0
	fourth	0	0
	fifth	0	0
	sixth	1	1
	seventh	0	0
	8th	0	0
	9th	1	1
	10th	0	0

Then click on the tab "Data" and then on "Solver", and the following pop-up window will appear.



In the frame "Objective", insert the cell $D$7, which contains the objective function.

Then check the option "Min" because it is a minimization problem.

In the frame "Variables" enter the area of the variables which is from A2 to U2, so $A$2:$U$2 and click the option "assume nonnegative"

In the frame "Constraints:" add the restrictions that have been mentioned above (task 1.1):

$A$2:$U$2 = binary

$G$7 = 1

$G$8 = 1

$G$9 = $I$9

$G$10 = $I$10

$G$11 = $I$11

$G$12:$G$17 = $I$12:$I$17

Last, in the frame "Engines" select the option "LP/Quadratic". The result is:

x12	x13	x14	x23	x26	x35	x36	x43	x45	x48	x410	x57	x58	x67	x69	x78	x79	x810	x910	x911	x1011
1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0

Z		Restrictions	
63	start	1	1
	final	1	1
	second	1	1
	third	0	0
	fourth	0	0
	fifth	0	0
	sixth	1	1
	seventh	0	0
	8th	0	0
	9th	1	1
	10th	0	0

task 1.2 1 → 2 → 6 → 9 → 11

Task 2:

Since node 6 represents a high-risk or hazardous area, it is desirable to avoid routes passing through it. To model this constraint within the optimization framework, prohibitively large edge weights are assigned to the connections involving node 6—specifically, a cost of 1400 is assigned to edge (6,7) and 1500 to edge (6,9). These inflated costs effectively discourage the inclusion of these edges in the optimal solution.

See the attached Excel file. The result is:

x12	x13	x14	x23	x26	x35	x36	x43	x45	x48	x410	x57	x58	x67	x69	x78	x79	x810	x910	x911	x1011
0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

Z		Restrictions	
70	start	1	1
	final	1	1
	second	0	0
	third	0	0
	fourth	1	1
	fifth	0	0
	sixth	0	0
	seventh	0	0
	8th	0	0
	9th	0	0
	10th	1	1

1 → 4 → 10 → 11