DIGITAL MATHEMATICS APPLIED IN DEFENCE AND SECURITY EDUCATION (DIMAS)

KA220-HED - Cooperation Partnerships in Higher Education

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Educational resource

(A3.2)

May 2025

SCENARIO No 3.5

Title	Transportation Problem
Short description	The transportation problem involves finding the optimal plan for transporting goods from various sources to designated destinations. Typically, the sources include industries, production units, or central supply warehouses, while the destinations consist of retail stores, regional warehouses, and similar outlets. Each source supplies a certain quantity of products or materials (supply), whereas each destination requires a specific amount (demand). In a typical transportation problem, the following data are given: • The contributions of each source • The destinations demand • The cost-benefit table between the nodes of the transmission network In the transportation problem, the primary goal is to determine the quantities to be transported between sources and destinations (the overall transportation plan) to optimize a cost-benefit function.
Topics Involved	**Logistics for Defence and Security**
Areas of Mathematics	• Linear Algebra • Mathematical Programming • Decision theory
Digital Mathematics Tools	Microsoft Excel – Solver tool
Learning Outcomes (knowledge, Skills, Responsibility & Autonomy)	**Knowledge:** Students know to: • define the correct variables • determine the objective function • identify any restrictions **Skills:** After passing through this scenario, the students will be able to: • model and simulate the transportation problem, • use software to solve the transportation problem, • work in a team to solve a problem, • can prepare a note and give a short presentation on the transportation problem solution. **Responsibility and Autonomy:** 1. Students are ready to critically evaluate his/her knowledge and recognise the importance of knowledge in solving transportation problems

	2. Take initiative and responsibility to propose an optimal solution to the transportation problem.
Methodologies adopted	Innovative teaching solutions will be used, such as specialized software to solve the transportation problem using the MS Excel Solver tool, as an element of decision-making support.
Prerequisites	• Mathematics: Linear Algebra • Mathematical Programming • Microsoft Excel
Estimated time	8 hours (including self-studies and syndicate work)

Task for students

TASK 1: There are two warehouses (with fuel) and two destinations (A, B). The fuel has to be transferred from the warehouses to the final destinations at the minimum cost. The following presents what the warehouses contain, what the final destinations require, and the cost per unit from a warehouse to the final destinations.

	A	B	**Supply**
1st	8	7	250
2nd	4	9	250
Demand	300	200	500

Please try to solve it by hand.

TASK 2:
2.1. Model the above problem from Task 1. Particularly:
- Define the correct variables
- Determine the objective function
- Identify any restrictions

2.2. Solve it using *MS Excel – Solver tool*

TASK 3: There are four warehouses (with fuel) and nine (9) destinations (A, B, C, D, E, F, G, H, and I). The fuel has to be transferred from the warehouses to the final destinations at the minimum cost.
The following presents what the warehouses contain, what the final destinations require, and the cost per unit from a warehouse to the final destinations.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**Supply**
1st	10	9	11	7	12	15	13	8	9	850
2nd	4	12	10	14	11	9	6	12	13	980
3rd	15	14	11	10	8	12	9	7	14	1050
4th	9	10	10	6	11	8	12	5	16	920
Demand	410	510	390	300	370	400	480	420	520	3800

	3.1. Model the above problem. Particularly: • Define the correct variables, • Determine the objective function • Identify any restrictions **3.2.** Solve it using *MS Excel – Solver tool* **TASK 4:** Solve the above problem (Task 3) with the condition the route from the 2nd warehouse to point G cannot be carried out.
Assessment	Observation: Students are evaluated during elaboration, in order to document their understanding of the model and solve the transportation problem, Project: Individual projects. Final presentation Complete initial and final feedback

Solution

Task 1:

From the 2nd warehouse, we will send 250 tons of fuel to destination A with a transportation cost of 4 x 250 = 1000 cost units.
Then, from the 1st warehouse, we will send 50 tons of fuel to destination A, with transportation costs of 8 x 50 = 400 cost units.
Finally, from the 1st warehouse, we will send 200 tons of fuel to destination B with a transportation cost of 7 x 200 = 1400 cost units.
So, the total cost is 1000 + 400 + 1400 = 2800

Task 2:
2.1

__Variables__
We define four decision variables, corresponding to the possible transportation routes. Let:

- x_1, x_2 represent the amounts of fuel (in tonnes) transported from Warehouse 1 to Destinations A and B, respectively.
- x_3, x_4 represent the amounts of fuel (in tonnes) transported from Warehouse 2 to Destinations A and B, respectively.

__Objective function__
The objective is to minimize the total transportation cost, represented by the function

$$Z = 8x_1 + 7x_2 + 4x_3 + 9x_4$$

where Z denotes the total cost in appropriate units.

Restrictions

For the supply:

$x_1 + x_2 = 250$ (The total fuel transported from Warehouse 1 is 250 tonnes)

$x_3 + x_4 = 250$ (The total fuel transported from Warehouse 1 is 250 tonnes)

For the demand:

$x_1 + x_3 = 300$ (Destination A requires a total of 300 tonnes of fuel)

$x_2 + x_4 = 200$ (Destination B requires a total of 300 tonnes of fuel)

$x_1, x_2, x_3, x_4 \geq 0$ (All transported quantities must be non-negative)

2.2. See the attached Excel file

Using an MS Excel File, enter in cells A1 to D1 the variables x1 to x4 as the figure below. In the corresponding cells A2 to D2, the values of the corresponding variables will be displayed.

In cell F1, please enter the variable Z1, while in cell F2, enter the following function

“Z2” = 8*A2+7*B2 + 4*C2 + 9*D2

In cells E6 to E9, please enter the restrictions as shown below:

“E6” = A2+B2

“E7” = C2+D2

“E8” = A2+C2

“E9” = B2 + D2

	A	B	C	D	E	F	G	H
1	x1	x2	x3	x4		Z		
2	50	200	250	0		2800		
3								
4								
5						Restrictions		
6					250		250	
7					250		250	
8					300		300	
9					200		200	
10								

Then click on the tab “Data” and then on “Solver”, and the following pop-up window will appear.

In the frame “Set Objective”, insert the cell **$F$2**, which contains the objective function.

Then check the option “**Min**” because it is a minimization problem.

In the frame “By Changing Variable Cells:” enter the area of the variables which is from A2 to D2, so **$A$2:$D$2**

In the frame “Subject to the Constraints:” add the four restrictions that have been mentioned above.

So:

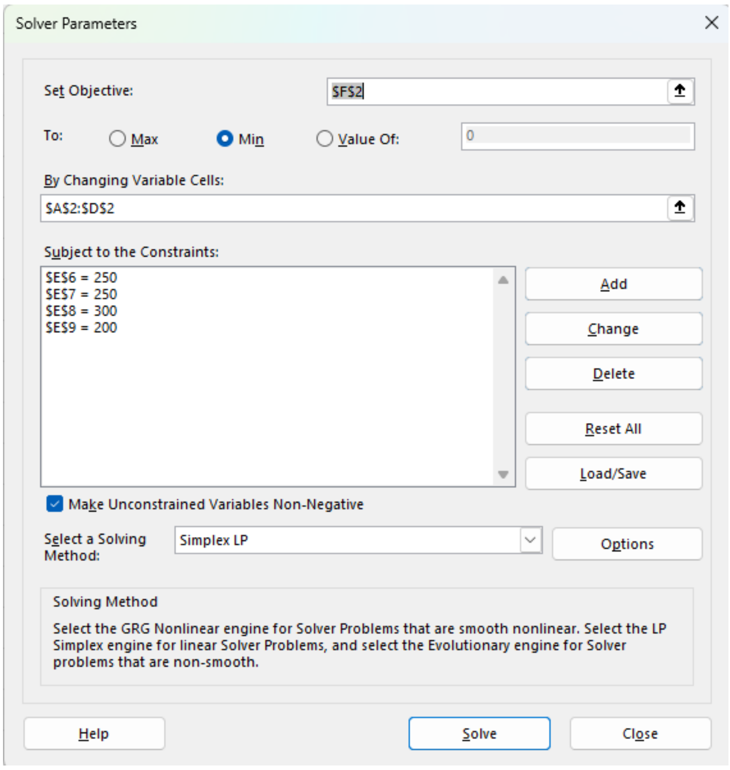
$E$6 = 250

$E$7 = 250

$E$8 = 300

$E$9 = 200

Last, in the frame “Select a Solving Method” select the option “**Simplex LP**”.



Then click the button “**Solve**” and the values will appear in the cells **A2** to **D2**.

Task 3:

3.1

Variables

We define thirty-six (36) decision variables, corresponding to the possible transportation routes. Let:

- $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$ represent the amounts of fuel (in tonnes) transported from Warehouse 1 to Destinations A, B, …, I, respectively.
- $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9$ represent the amounts of fuel (in tonnes) transported from Warehouse 2 to Destinations A, B, …, I, respectively.
- $w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9$ represent the amounts of fuel (in tonnes) transported from Warehouse 3 to Destinations A, B, …, I, respectively.
- $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9$ represent the amounts of fuel (in tonnes) transported from Warehouse 4 to Destinations A, B, …, I, respectively.

Objective function

The objective is to minimize the total transportation cost, represented by the function

$Z = 10x_1 + 9x_2 + 11x_3 + 7x_4 + 12x_5 + 15x_6 + 13x_7 + 8x_8 + 9x_9 + 4y_1 + 12y_2 + 10y_3 + 14y_4 + 11y_5 + 9y_6 + 6y_7 + 12y_8 + 13y_9 + 15w_1 + 14w_2 + 11w_3 + 10w_4 + 8w_5 + 12w_6 + 9w_7 + 14w_9 + 9u_1 + 10u_2 + 10u_3 + 6u_4 + 11u_5 + 8u_6 + 12u_7 + 5u_8 + 16u_9$

where Z denotes the total cost in appropriate units.

<u>Restrictions</u>

For the supply:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 = 850$

(The total fuel transported from Warehouse 1 is 850 tonnes)

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 = 980$

(The total fuel transported from Warehouse 1 is 980 tonnes)

$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_9 = 1050$

(The total fuel transported from Warehouse 3 is 1050 tonnes)

$u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 + u_9 = 920$

(The total fuel transported from Warehouse 4 is 920 tonnes)

For the demand:

$x_1 + y_1 + w_1 + u_1 = 410$ (Destination A requires a total of 410 tonnes of fuel)

$x_2 + y_2 + w_2 + u_2 = 510$ (Destination B requires a total of 510 tonnes of fuel)

$x_3 + y_3 + w_3 + u_3 = 390$ (Destination C requires a total of 390 tonnes of fuel)

$x_4 + y_4 + w_4 + u_4 = 300$ (Destination D requires a total of 300 tonnes of fuel)

$x_5 + y_5 + w_5 + u_5 = 370$ (Destination E requires a total of 370 tonnes of fuel)

$x_6 + y_6 + w_6 + u_6 = 400$ (Destination F requires a total of 400 tonnes of fuel)

$x_7 + y_7 + w_7 + u_7 = 480$ (Destination G requires a total of 480 tonnes of fuel)

$x_8 + y_8 + w_8 + u_8 = 420$ (Destination H requires a total of 420 tonnes of fuel)

$x_9 + y_9 + w_9 + u_9 = 520$ (Destination I requires a total of 520 tonnes of fuel)

$x_i, y_i, w_i, u_i \geq 0, \forall i = 1,2,\ldots,9$ (All transported quantities must be non-negative)

3.2. See the attached Excel file

Task 4:

See the attached Excel file